Uncertainty in pressure-based velocity probes

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David M. Birch
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Consider a conventional Pitot-static probe operating in low subsonic conditions, using a differential pressure transducer. The ith reading of the transducer will be related to the local velocity according to the relationship

$$(P_0 - P_i) = \frac{1}{2}\rho U_i^2, \tag{1}$$

where P is the local static pressure, P_0 is the stagnation pressure (assumed invariant, so that the difference $P_0 - P_i$ may be directly read from a differential pressure transducer between the static and stagnation lines), and U is the local mean velocity. If we then consider a case when the pressure changes from some state 1 to some other state 2, (1) may be re-written as

$$\left(P_0 - P_2\right) - \left(P_0 - P_1\right) = \frac{1}{2}\rho U_2^2 - \frac{1}{2}\rho U_1^2. \tag{2}$$

The stagnation pressure may then be eliminated, and (2) re-arranged to yield

$$\frac{2}{\rho}\left(P_1 - P_2\right) = U_2^2 - U_1^2. \tag{3}$$

The difference on the right-hand side may then be expanded, so that

$$\begin{aligned} -\frac{2}{\rho}\Delta P &= \Delta U\left(U_2 + U_1\right) \\ &= \Delta U\left(U_2 - U_1 + 2U_1\right) \end{aligned} \tag{4}$$

where the Δ operator represents the signed difference in any quantity moving from state 1 to state 2. Simplifying,

$$-\frac{2}{\rho}\Delta P = \Delta U\left(\Delta U + 2U_1\right). \tag{5}$$

If it is assumed that the change is due to some small uncertainty in pressure, the differences ΔP and ΔU may be expressed instead as dimensional uncertainties ϵ_P and ϵ_U, respectively. Then, (5) may be expressed as

$$\epsilon_U^2 + 2U\epsilon_U + \frac{2}{\rho}\epsilon_P = 0 \tag{6}$$

This quadratic in ϵ_U admits the solutions

$$\epsilon_U = \sqrt{U^2 + \frac{2}{\rho}\epsilon_P} - U \tag{7}$$

noting that the negative root corresponds to a decrease in velocity with increasing pressure, which is nonphysical. Since the quantity of interest is typically the full-scale uncertainty of the measurement system, (7) may be normalized against some full-scale range U_{FS} as

$$\frac{\epsilon_U}{U_{FS}} = \sqrt{\left(\frac{U}{U_{FS}}\right)^2 + \frac{2\epsilon_P}{\rho U_{FS}^2}} - \frac{U}{U_{FS}} \tag{8}$$

If the pressure sensor was selected such that the full-scale differential range P_{FS} matched the stagnation pressure at U_{FS} (or, alternatively, taking U_{FS} as the maximum measurable velocity), then, substituting (1),

$$\frac{\epsilon_U}{U_{FS}} = \sqrt{\left(\frac{U}{U_{FS}}\right)^2 + \frac{\epsilon_P}{P_{FS}}} - \frac{U}{U_{FS}} \tag{9}$$

For typical pressure sensors, the uncertainty quoted by the manufacturer is invariant; then, ϵ_P/P_{FS} is constant and specified. Equation (9) then provides a universal relationship between the uncertainty of the velocity probe and the velocity being measured. The relationship shown in (9) is illustrated graphically in figure 1.

Since the maximum uncertainty occurs when $U/U_{FS} = 0$, this value can be substituted into (9) to obtain

$$\max\left(\frac{\epsilon_U}{U_{FS}}\right) = \sqrt{\frac{\epsilon_P}{P_{FS}}}, \tag{10}$$

This relationship is plotted in Figure 2, and demonstrates the difficulty of obtaining high-quality velocity data from pressure-based measurements at low speeds. A 0.5% FS total error band is typical for high-quality differential pressure sensors; with this uncertainty, the lowest detectable velocity will be 7.1% of the maximum measurable velocity.

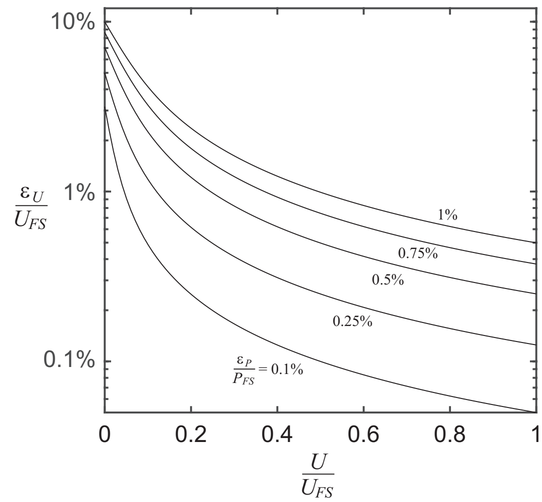

Figure 1: Relationship between uncertainty and velocity for a standard Pitot-static probe.

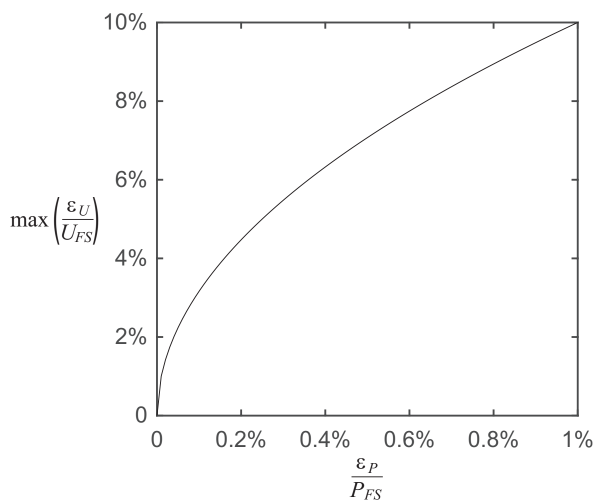

Figure 2: Relationship between maximum uncertainty in velocity and uncertainty in pressure.